MULTIMEDIA UNIVERSITY

STUDENT ID NO

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# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 3, 2022/2023**

## BMS2024 - ADVANCED MANAGERIAL STATISTICS

(All Sections/Groups)

5 OCTOBER 2023
9.00 a.m. – 11.00 a.m.
( 2 Hours )

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## INSTRUCTIONS TO STUDENT

1. This question paper consist of **TEN (10) pages** (excluding the cover page).
2. Answer **ALL** questions. The marks distributions are given in parentheses.
3. Students are allowed to use non-programmable scientific calculators (without restriction).
4. Please write all your answers in the Answer Booklet provided. All necessary workings **MUST** be shown.
5. Statistical formulas and tables are provided in the appendix.

## Question 1 [25 marks]

The supervisor of a streaming service platform that offers one type of streaming content and one subscription plan at the most budget-friendly rate wishes to gain deeper insights into viewership patterns during weekday working hours. Data has been collected on the specific content chosen by 500 subscribers and their preferred subscription plan, and this information has been organized into the following table. Subscribers can choose one type of content from Movies, TV Series, Documentaries, and News, and they can also select one subscription plan from Basic, Standard, and Premium.

| Subscription Plan | Type of Content | | | | |
|---|---|---|---|---|---|
| | Movies | TV Series | Documentaries | News | Total |
| Basic | 45 | 8 | 12 | 15 | 80 |
| Standard | 98 | 15 | 32 | 16 | 161 |
| Premium | 80 | 84 | 54 | 41 | 259 |
| Total | 223 | 107 | 98 | 72 | 500 |

The supervisor is interested in understanding the relationship between the type of content chosen and the selected subscription plan.

(a) Formulate the hypotheses for the above problem.

**(4 marks)**

(b) At the 0.05 percent significance level, is there any statistically significant evidence of a connection between the chosen type of content and the selected subscription plan?

**(21 marks)**

## Question 2 [25 marks]

(a) Differentiate between the Kruskal-Wallis rank sum test and one-way analysis of variance in terms of their assumptions and the circumstances under which each should be applied. Explain.

**(5 marks)**

**Continued...**

(b) In a research project examining smartphone usage, 17 participants were surveyed to determine their screen time on three different mobile apps (Instagram, TikTok, and Facebook). The study aimed to ascertain if there is any significant difference in the median screen time across these four apps at a significance level of 0.01.

| Instagram | TikTok | Facebook |
|---|---|---|
| 14 | 18 | 5 |
| 7 | 19 | 9 |
| 8 | 14 | 2 |
| 5 | 7 | 7 |
| 3 | 9 | 3 |
| 17 | 12 | |

**(20 marks)**

**Question 3 [25 marks]**

TechGizmo Magazine compiles product reviews for various gadgets on the market. These reviews encompass ratings for features, design, performance, and retail price for thirty different tech products. The objective of the regression model is to predict the retail price based on a variable that represents the combined ratings for features, design, and performance. The data has been examined, and the outcomes are summarized in the table below:

| *Regression Statistics* | |
|---|---|
| Multiple R | 0.9453 |
| R Square | 0.9525 |
| Adjusted R Square | 0.8910 |
| Standard Error | 0.2347 |
| Observations | 30 |

ANOVA

| | *df* | *SS* | *MS* | *F* | *Significance F* |
|---|---|---|---|---|---|
| Regression | 3 | 12.0268 | 4.0089 | 75.56712 | 8.88341E-13 |
| Residual | 26 | 1.4318 | 0.0551 | | |
| Total | 29 | 13.4586 | | | |

| | *Coefficients* | *Std Error* | *t stat* | *P-value* |
|---|---|---|---|---|
| Intercept | 7.5891 | 2.4450 | 3.1039 | 0.00457 |
| Features | –2.3577 | 0.6379 | –3.6958 | 0.00115 |
| Design | 1.6122 | 0.2954 | 5.4586 | 0.00003 |
| Performance | 0.5012 | 0.1259 | 3.9814 | 0.00022 |

**Continued...**

a) What is multiple linear regression analysis? Explain. **(3 marks)**

b) State the multiple linear regression equation for the above data. **(2 marks)**

c) Interpret the values of the coefficient of determination and the adjusted coefficient of determination. Explain why the values differ. **(4 marks)**

d) At the 1 percent level of significance, test if the regression model is significant. Use the p-value approach. **(5 marks)**

e) At the 1 percent level of significance, test if each independent variable have a significant contribution to the model. Use the p-value approach. **(5 marks)**

f) For the independent variable(s) which is (are) found to have a significant contribution to the model, interpret the extent of the contribution. **(6 marks)**

**Continued...**

**Question 4 [25 marks]**

A group of architects is developing a series of building materials for construction projects, and they aim to identify the most suitable material for different architectural styles: modern, minimalist, and industrial. To achieve this, they conducted tests by using each of the two materials on one prototype structure for each architectural style. The performance results for these 24 different combinations are outlined in table below, followed by the Two-way ANOVA output from EXCEL.

**Architectural styles**

| Material/ Styles | Modern | Minimalist | Industrial |
|---|---|---|---|
| Material 1 | 85 | 92 | 99 |
| | 88 | 87 | 99 |
| | 77 | 83 | 78 |
| | 74 | 86 | 74 |
| Material 2 | 73 | 76 | 89 |
| | 76 | 83 | 82 |
| | 95 | 73 | 83 |
| | 82 | 71 | 79 |

**Two-way ANOVA output from EXCEL**

ANOVA

| *Source of Variation* | *SS* | *df* | *MS* | *F* | *P-value* | *F crit* |
|---|---|---|---|---|---|---|
| Sample | 150 | 1 | 150 | 2.39043825 | 0.13948004 | 4.41387342 |
| Columns | 88.0833333 | 2 | 44.0416667 | 0.70185923 | 0.5087306 | 3.55455715 |
| Interaction | 139.75 | 2 | 69.875 | 1.11354582 | 0.34998672 | 3.55455715 |
| Within | 1129.5 | 18 | 62.75 | | | |
| Total | 1507.33333 | 23 | | | | |

At the 0.05 level of significance:

a. Test if there is an interaction between architectural styles and material? **(7 marks)**

b. Test if there is an effect due to architectural style. **(4 marks)**

c. Test if there is an effect due to the material? **(4 marks)**

d. Calculate the average values and plot the estimated value of the performance results for these different combinations? **(8 marks)**

e. Discuss the results obtained in (a), (b), (c), and (d) above? **(2 marks)**

**End of Questions**

# STATISTICAL FORMULAE

## A. DESCRIPTIVE STATISTICS

Sample Mean = $\overline{X} = \dfrac{\sum_{i=1}^{n} X_i}{n}$ Sample Standard Deviation = s = $\sqrt{\dfrac{\sum_{i=1}^{n} X_i^2}{n-1} - \dfrac{\left(\sum_{i=1}^{n} X_i\right)^2}{n(n-1)}}$

*where* $n$ = *number of observations*
$X_i$ = *the* $i^{th}$ *observation of the data*

## B. HYPOTHESIS TESTING

| **Types of Error** |
|---|
| Type I Error = $\alpha$= P(Rejecting $H_0$ \| $H_0$ is true) where, Confidence Interval = 1 - $\alpha$ |
| Type II Error = $\beta$= P(Not Rejecting $H_0$ \| $H_0$ is false) |

| **One Sample Mean Test** | |
|---|---|
| **σ Known** | **σ Unknown** |
| $z = \dfrac{\overline{x} - \mu}{\sigma/\sqrt{n}}$ | $t = \dfrac{\overline{x} - \mu}{s/\sqrt{n}}$ |

| **Two Sample Mean Test** | |
|---|---|
| **Comparing Means for Two Independent Populations** | |
| **[Standard Deviation (σ) Known]** $z = \dfrac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$ | **[Standard Deviation (σ) Not Known]** $t = \dfrac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(1/n_1 + 1/n_2\right)}}$ where $S_p^2 = \dfrac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1+n_2-2)}$ |

| **Two Sample Mean Test** |
|---|
| **Comparing Means for Two Paired Populations** |
| $t = \dfrac{(\overline{D} - \mu_D)}{S_D/\sqrt{n}}$ where $\overline{D} = \dfrac{\sum_{i=1}^{n} D_i}{n}$ and $S_D = \sqrt{\dfrac{\sum_{i=1}^{n} D_i^2}{n-1} - \dfrac{\left(\sum_{i=1}^{n} D_i\right)^2}{n(n-1)}}$ |

**Continued...**

**Non-Parametric Analysis**

**Wilcoxon Rank Sum Test**

$$Z = \frac{\left(T_1 - \mu_{T_1}\right)}{\sigma_{T_1}}$$ where

$$\mu_{T1} = \frac{n_1(n+1)}{2}$$ and

$$\sigma_{T_1} = \sqrt{\frac{n_1 n_2 (n+1)}{12}}$$ where $n = n_1 + n_2$

**Wilcoxon Signed Rank Sum Test**

$$Z = \frac{\left(T_+ - \mu_{T_+}\right)}{\sigma_{T_+}}$$ where

$$\mu_{T+} = \frac{n(n+1)}{4}$$ and

$$\sigma_{T_+} = \sqrt{\frac{n(n+1)(2n+1)}{24}}$$

**Kruskal-Wallis Rank Test**

$$H = \left[\frac{12}{n(n+1)}\sum_{j=1}^{c}\frac{T_j^2}{n_j}\right] - 3(n+1)$$ where the critical value is $\chi^2$ with $df = c - 1$

*Check ranking sum: $\sum T_j = n(n+1)/2$*

**Chi-Square Test**

$$\chi^2 = \sum^{n}\frac{(O-E)^2}{E}$$

*where O = Frequency of Observed Values and E = Frequency of Expected Values*

*with df = c – 1 where c = number of categories*
*or*
*with df = (r – 1)(c – 1) where r = number of rows and c = number of columns*

## C. ANALYSIS OF VARIANCE (ANOVA)

**One-Way ANOVA**

| Source | Degrees of Freedom | Sum of Squares | Mean Squares | F-statistic |
|---|---|---|---|---|
| Among Groups | *c - 1* | SSA | MSA = SSA/*c-1* | MSA/MSW |
| Within Groups | *n - c* | SSW | MSW = SSW/*n-c* | |
| Total | *n - 1* | SST | | |

$$SST = \sum_{j=1}^{c}\sum_{i=1}^{n_j}\left(X_{ij} - \overline{\overline{X}}\right)^2$$ *or alternative formula:* $$SST = \left(\sum_{j=1}^{c}\sum_{i=1}^{n_j}X_{ij}^2\right) - \frac{\left(\sum_{j=1}^{c}\sum_{i=1}^{n_j}X_{ij}\right)^2}{n}$$

$$SSA = \sum_{j=1}^{c} n_j\left(\overline{X}_j - \overline{\overline{X}}\right)^2$$ *and* $SSW = SST - SSA$

*where n = number of observations, c = number of groups and* $\overline{\overline{X}}$ *= overall mean*

**Continued...**

**Tukey-Kramer Procedure**

$$\text{Critical Range} = Q_U \sqrt{\frac{MSW}{2}\left[\frac{1}{n_i}+\frac{1}{n_j}\right]}$$

*where $Q_u$ = the upper tail critical value from a Studentized Range Distribution having (c) degrees of freedom in the numerator and (n – c) degrees of freedom in the denominator at a given level of significance*

**Two-Way ANOVA**

| Source | Degrees of Freedom | Sum of Squares | Mean Squares | F-statistic |
|---|---|---|---|---|
| A | $r-1$ | SSA | MSA = SSA/$(r-1)$ | MSA / MSE |
| B | $c-1$ | SSB | MSB = SSB/$(c-1)$ | MSB / MSE |
| AB | $(r-1)(c-1)$ | SSAB | MSAB = SSAB/$(r-1)(c-1)$ | MSAB / MSE |
| Error | $rc\,(n-1)$ | SSE | MSE = SSE/$rc(n'-1)$ | |
| Total | $n-1$ | SST | | |

*where,*

*Factor A levels are represented by the rows and Factor B levels are represented by the columns and*
*n = number of observations*
*c = number of columns*
*r = number of rows*
*n' = number of replicates*

$$SST = \sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{n'}\left(X_{ijk}-\overline{\overline{X}}\right)^2 \qquad SSA = cn'\sum_{i=1}^{r}\left(\overline{X}_i-\overline{\overline{X}}\right)^2$$

$$SSB = rn'\sum_{j=1}^{c}\left(\overline{X}j-\overline{\overline{X}}\right)^2 \qquad \text{where } \overline{\overline{X}} = \textit{overall mean}$$

$$SSE = (n'-1)\left[S_1^2+S_2^2+\ldots\ldots+S_k^2\right] \qquad \text{where } S_i^2 = \text{variance of each block}$$

## D. REGRESSION ANALYSIS

**Multiple Linear Regression**

Population Model: $Y = \beta_0+\beta_1X_1+\beta_2X_2+\ldots\ldots+\beta_kX_k+\varepsilon$

Sample Model: $\hat{y} = b_0+b_1x_1+b_2x_2+\ldots\ldots+b_kx_k+e$

Adjusted R-Square $= 1-\left[\dfrac{(1-R^2)(n-1)}{(n-p-1)}\right]$ where $p$ = number of independent/predictor variables

**Continued...**

| **ANOVA Table for Regression** | | | |
|---|---|---|---|
| **Source** | **Degrees of Freedom** | **Sum of Squares** | **Mean Squares** |
| Regression | $p$ | SSR | MSR = SSR/$p$ |
| Error/Residual | $n-p-1$ | SSE | MSE = SSE/$(n-p-1)$ |
| Total | $n-1$ | SST | |

**Test Statistic for Significance of the Overall Regression Model**

F = MSR/MSE

**Test Statistic for Significance of Each Predictor Variable**

$t_i = \dfrac{b_i}{S_{b_i}}$ and the critical value = $\pm t_{\alpha/2,(n-p-1)}$

**Continued...**

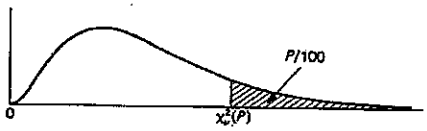
# TABLE 8. PERCENTAGE POINTS OF THE $\chi^2$-DISTRIBUTION

This table gives percentage points $\chi^2_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{2^{\frac{1}{2}\nu}\,\Gamma\left(\frac{\nu}{2}\right)} \int_{\chi^2_\nu(P)}^{\infty} x^{\frac{1}{2}\nu - 1} e^{-\frac{1}{2}x}\, dx.$$

If $X$ is a variable distributed as $\chi^2$ with $\nu$ degrees of freedom, $P/100$ is the probability that $X \geqslant \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu - 1}$ and unit variance.



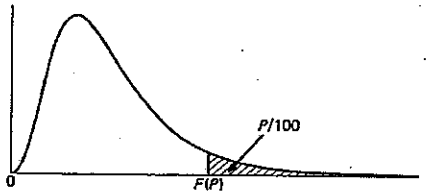

(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

| $P$ | 50 | 40 | 30 | 20 | 10 | 5 | 2·5 | 1 | 0·5 | 0·1 | 0·05 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\nu = 1$ | 0·4549 | 0·7083 | 1·074 | 1·642 | 2·706 | 3·841 | 5·024 | 6·635 | 7·879 | 10·83 | 12·12 |
| 2 | 1·386 | 1·833 | 2·408 | 3·219 | 4·605 | 5·991 | 7·378 | 9·210 | 10·60 | 13·82 | 15·20 |
| 3 | 2·366 | 2·946 | 3·665 | 4·642 | 6·251 | 7·815 | 9·348 | 11·34 | 12·84 | 16·27 | 17·73 |
| 4 | 3·357 | 4·045 | 4·878 | 5·989 | 7·779 | 9·488 | 11·14 | 13·28 | 14·86 | 18·47 | 20·00 |
| 5 | 4·351 | 5·132 | 6·064 | 7·289 | 9·236 | 11·07 | 12·83 | 15·09 | 16·75 | 20·52 | 22·11 |
| 6 | 5·348 | 6·211 | 7·231 | 8·558 | 10·64 | 12·59 | 14·45 | 16·81 | 18·55 | 22·46 | 24·10 |
| 7 | 6·346 | 7·283 | 8·383 | 9·803 | 12·02 | 14·07 | 16·01 | 18·48 | 20·28 | 24·32 | 26·02 |
| 8 | 7·344 | 8·351 | 9·524 | 11·03 | 13·36 | 15·51 | 17·53 | 20·09 | 21·95 | 26·12 | 27·87 |
| 9 | 8·343 | 9·414 | 10·66 | 12·24 | 14·68 | 16·92 | 19·02 | 21·67 | 23·59 | 27·88 | 29·67 |
| 10 | 9·342 | 10·47 | 11·78 | 13·44 | 15·99 | 18·31 | 20·48 | 23·21 | 25·19 | 29·59 | 31·42 |
| 11 | 10·34 | 11·53 | 12·90 | 14·63 | 17·28 | 19·68 | 21·92 | 24·72 | 26·76 | 31·26 | 33·14 |
| 12 | 11·34 | 12·58 | 14·01 | 15·81 | 18·55 | 21·03 | 23·34 | 26·22 | 28·30 | 32·91 | 34·82 |
| 13 | 12·34 | 13·64 | 15·12 | 16·98 | 19·81 | 22·36 | 24·74 | 27·69 | 29·82 | 34·53 | 36·48 |
| 14 | 13·34 | 14·69 | 16·22 | 18·15 | 21·06 | 23·68 | 26·12 | 29·14 | 31·32 | 36·12 | 38·11 |
| 15 | 14·34 | 15·73 | 17·32 | 19·31 | 22·31 | 25·00 | 27·49 | 30·58 | 32·80 | 37·70 | 39·72 |
| 16 | 15·34 | 16·78 | 18·42 | 20·47 | 23·54 | 26·30 | 28·85 | 32·00 | 34·27 | 39·25 | 41·31 |
| 17 | 16·34 | 17·82 | 19·51 | 21·61 | 24·77 | 27·59 | 30·19 | 33·41 | 35·72 | 40·79 | 42·88 |
| 18 | 17·34 | 18·87 | 20·60 | 22·76 | 25·99 | 28·87 | 31·53 | 34·81 | 37·16 | 42·31 | 44·43 |
| 19 | 18·34 | 19·91 | 21·69 | 23·90 | 27·20 | 30·14 | 32·85 | 36·19 | 38·58 | 43·82 | 45·97 |
| 20 | 19·34 | 20·95 | 22·77 | 25·04 | 28·41 | 31·41 | 34·17 | 37·57 | 40·00 | 45·31 | 47·50 |
| 21 | 20·34 | 21·99 | 23·86 | 26·17 | 29·62 | 32·67 | 35·48 | 38·93 | 41·40 | 46·80 | 49·01 |
| 22 | 21·34 | 23·03 | 24·94 | 27·30 | 30·81 | 33·92 | 36·78 | 40·29 | 42·80 | 48·27 | 50·51 |
| 23 | 22·34 | 24·07 | 26·02 | 28·43 | 32·01 | 35·17 | 38·08 | 41·64 | 44·18 | 49·73 | 52·00 |
| 24 | 23·34 | 25·11 | 27·10 | 29·55 | 33·20 | 36·42 | 39·36 | 42·98 | 45·56 | 51·18 | 53·48 |
| 25 | 24·34 | 26·14 | 28·17 | 30·68 | 34·38 | 37·65 | 40·65 | 44·31 | 46·93 | 52·62 | 54·95 |
| 26 | 25·34 | 27·18 | 29·25 | 31·79 | 35·56 | 38·89 | 41·92 | 45·64 | 48·29 | 54·05 | 56·41 |
| 27 | 26·34 | 28·21 | 30·32 | 32·91 | 36·74 | 40·11 | 43·19 | 46·96 | 49·64 | 55·48 | 57·86 |
| 28 | 27·34 | 29·25 | 31·39 | 34·03 | 37·92 | 41·34 | 44·46 | 48·28 | 50·99 | 56·89 | 59·30 |
| 29 | 28·34 | 30·28 | 32·46 | 35·14 | 39·09 | 42·56 | 45·72 | 49·59 | 52·34 | 58·30 | 60·73 |
| 30 | 29·34 | 31·32 | 33·53 | 36·25 | 40·26 | 43·77 | 46·98 | 50·89 | 53·67 | 59·70 | 62·16 |
| 32 | 31·34 | 33·38 | 35·66 | 38·47 | 42·58 | 46·19 | 49·48 | 53·49 | 56·33 | 62·49 | 65·00 |
| 34 | 33·34 | 35·44 | 37·80 | 40·68 | 44·90 | 48·60 | 51·97 | 56·06 | 58·96 | 65·25 | 67·80 |
| 36 | 35·34 | 37·50 | 39·92 | 42·88 | 47·21 | 51·00 | 54·44 | 58·62 | 61·58 | 67·99 | 70·59 |
| 38 | 37·34 | 39·56 | 42·05 | 45·08 | 49·51 | 53·38 | 56·90 | 61·16 | 64·18 | 70·70 | 73·35 |
| 40 | 39·34 | 41·62 | 44·16 | 47·27 | 51·81 | 55·76 | 59·34 | 63·69 | 66·77 | 73·40 | 76·09 |
| 50 | 49·33 | 51·89 | 54·72 | 58·16 | 63·17 | 67·50 | 71·42 | 76·15 | 79·49 | 86·66 | 89·56 |
| 60 | 59·33 | 62·13 | 65·23 | 68·97 | 74·40 | 79·08 | 83·30 | 88·38 | 91·95 | 99·61 | 102·7 |
| 70 | 69·33 | 72·36 | 75·69 | 79·71 | 85·53 | 90·53 | 95·02 | 100·4 | 104·2 | 112·3 | 115·6 |
| 80 | 79·33 | 82·57 | 86·12 | 90·41 | 96·58 | 101·9 | 106·6 | 112·3 | 116·3 | 124·8 | 128·3 |
| 90 | 89·33 | 92·76 | 96·52 | 101·1 | 107·6 | 113·1 | 118·1 | 124·1 | 128·3 | 137·2 | 140·8 |
| 100 | 99·33 | 102·9 | 106·9 | 111·7 | 118·5 | 124·3 | 129·6 | 135·8 | 140·2 | 149·4 | 153·2 |

**Continued...**

# TABLE 12(*b*). 5 PER CENT POINTS OF THE *F*-DISTRIBUTION

If $F = \frac{X_1}{\nu_1} \Big/ \frac{X_2}{\nu_2}$, where $X_1$ and $X_2$ are independent random variables distributed as $\chi^2$ with $\nu_1$ and $\nu_2$ degrees of freedom respectively, then the probabilities that $F \geqslant F(P)$ and that $F \leqslant F'(P)$ are both equal to $P/100$. Linear interpolation in $\nu_1$ and $\nu_2$ will generally be sufficiently accurate except when either $\nu_1 > 12$ or $\nu_2 > 40$, when harmonic interpolation should be used.




(This shape applies only when $\nu_1 \geqslant 3$. When $\nu_1 < 3$ the mode is at the origin.)

| $\nu_2 =$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 10 | 12 | 24 | ∞ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\nu_2 = 1$ | 161·4 | 199·5 | 215·7 | 224·6 | 230·2 | 234·0 | 236·8 | 238·9 | 241·9 | 243·9 | 249·1 | 254·3 |
| 2 | 18·51 | 19·00 | 19·16 | 19·25 | 19·30 | 19·33 | 19·35 | 19·37 | 19·40 | 19·41 | 19·45 | 19·50 |
| 3 | 10·13 | 9·552 | 9·277 | 9·117 | 9·013 | 8·941 | 8·887 | 8·845 | 8·786 | 8·745 | 8·639 | 8·526 |
| 4 | 7·709 | 6·944 | 6·591 | 6·388 | 6·256 | 6·163 | 6·094 | 6·041 | 5·964 | 5·912 | 5·774 | 5·628 |
| 5 | 6·608 | 5·786 | 5·409 | 5·192 | 5·050 | 4·950 | 4·876 | 4·818 | 4·735 | 4·678 | 4·527 | 4·365 |
| 6 | 5·987 | 5·143 | 4·757 | 4·534 | 4·387 | 4·284 | 4·207 | 4·147 | 4·060 | 4·000 | 3·841 | 3·669 |
| 7 | 5·591 | 4·737 | 4·347 | 4·120 | 3·972 | 3·866 | 3·787 | 3·726 | 3·637 | 3·575 | 3·410 | 3·230 |
| 8 | 5·318 | 4·459 | 4·066 | 3·838 | 3·687 | 3·581 | 3·500 | 3·438 | 3·347 | 3·284 | 3·115 | 2·928 |
| 9 | 5·117 | 4·256 | 3·863 | 3·633 | 3·482 | 3·374 | 3·293 | 3·230 | 3·137 | 3·073 | 2·900 | 2·707 |
| 10 | 4·965 | 4·103 | 3·708 | 3·478 | 3·326 | 3·217 | 3·135 | 3·072 | 2·978 | 2·913 | 2·737 | 2·538 |
| 11 | 4·844 | 3·982 | 3·587 | 3·357 | 3·204 | 3·095 | 3·012 | 2·948 | 2·854 | 2·788 | 2·609 | 2·404 |
| 12 | 4·747 | 3·885 | 3·490 | 3·259 | 3·106 | 2·996 | 2·913 | 2·849 | 2·753 | 2·687 | 2·505 | 2·296 |
| 13 | 4·667 | 3·806 | 3·411 | 3·179 | 3·025 | 2·915 | 2·832 | 2·767 | 2·671 | 2·604 | 2·420 | 2·206 |
| 14 | 4·600 | 3·739 | 3·344 | 3·112 | 2·958 | 2·848 | 2·764 | 2·699 | 2·602 | 2·534 | 2·349 | 2·131 |
| 15 | 4·543 | 3·682 | 3·287 | 3·056 | 2·901 | 2·790 | 2·707 | 2·641 | 2·544 | 2·475 | 2·288 | 2·066 |
| 16 | 4·494 | 3·634 | 3·239 | 3·007 | 2·852 | 2·741 | 2·657 | 2·591 | 2·494 | 2·425 | 2·235 | 2·010 |
| 17 | 4·451 | 3·592 | 3·197 | 2·965 | 2·810 | 2·699 | 2·614 | 2·548 | 2·450 | 2·381 | 2·190 | 1·960 |
| 18 | 4·414 | 3·555 | 3·160 | 2·928 | 2·773 | 2·661 | 2·577 | 2·510 | 2·412 | 2·342 | 2·150 | 1·917 |
| 19 | 4·381 | 3·522 | 3·127 | 2·895 | 2·740 | 2·628 | 2·544 | 2·477 | 2·378 | 2·308 | 2·114 | 1·878 |
| 20 | 4·351 | 3·493 | 3·098 | 2·866 | 2·711 | 2·599 | 2·514 | 2·447 | 2·348 | 2·278 | 2·082 | 1·843 |
| 21 | 4·325 | 3·467 | 3·072 | 2·840 | 2·685 | 2·573 | 2·488 | 2·420 | 2·321 | 2·250 | 2·054 | 1·812 |
| 22 | 4·301 | 3·443 | 3·049 | 2·817 | 2·661 | 2·549 | 2·464 | 2·397 | 2·297 | 2·226 | 2·028 | 1·783 |
| 23 | 4·279 | 3·422 | 3·028 | 2·796 | 2·640 | 2·528 | 2·442 | 2·375 | 2·275 | 2·204 | 2·005 | 1·757 |
| 24 | 4·260 | 3·403 | 3·009 | 2·776 | 2·621 | 2·508 | 2·423 | 2·355 | 2·255 | 2·183 | 1·984 | 1·733 |
| 25 | 4·242 | 3·385 | 2·991 | 2·759 | 2·603 | 2·490 | 2·405 | 2·337 | 2·236 | 2·165 | 1·964 | 1·711 |
| 26 | 4·225 | 3·369 | 2·975 | 2·743 | 2·587 | 2·474 | 2·388 | 2·321 | 2·220 | 2·148 | 1·946 | 1·691 |
| 27 | 4·210 | 3·354 | 2·960 | 2·728 | 2·572 | 2·459 | 2·373 | 2·305 | 2·204 | 2·132 | 1·930 | 1·672 |
| 28 | 4·196 | 3·340 | 2·947 | 2·714 | 2·558 | 2·445 | 2·359 | 2·291 | 2·190 | 2·118 | 1·915 | 1·654 |
| 29 | 4·183 | 3·328 | 2·934 | 2·701 | 2·545 | 2·432 | 2·346 | 2·278 | 2·177 | 2·104 | 1·901 | 1·638 |
| 30 | 4·171 | 3·316 | 2·922 | 2·690 | 2·534 | 2·421 | 2·334 | 2·266 | 2·165 | 2·092 | 1·887 | 1·622 |
| 32 | 4·149 | 3·295 | 2·901 | 2·668 | 2·512 | 2·399 | 2·313 | 2·244 | 2·142 | 2·070 | 1·864 | 1·594 |
| 34 | 4·130 | 3·276 | 2·883 | 2·650 | 2·494 | 2·380 | 2·294 | 2·225 | 2·123 | 2·050 | 1·843 | 1·569 |
| 36 | 4·113 | 3·259 | 2·866 | 2·634 | 2·477 | 2·364 | 2·277 | 2·209 | 2·106 | 2·033 | 1·824 | 1·547 |
| 38 | 4·098 | 3·245 | 2·852 | 2·619 | 2·463 | 2·349 | 2·262 | 2·194 | 2·091 | 2·017 | 1·808 | 1·527 |
| 40 | 4·085 | 3·232 | 2·839 | 2·606 | 2·449 | 2·336 | 2·249 | 2·180 | 2·077 | 2·003 | 1·793 | 1·509 |
| 60 | 4·001 | 3·150 | 2·758 | 2·525 | 2·368 | 2·254 | 2·167 | 2·097 | 1·993 | 1·917 | 1·700 | 1·389 |
| 120 | 3·920 | 3·072 | 2·680 | 2·447 | 2·290 | 2·175 | 2·087 | 2·016 | 1·910 | 1·834 | 1·608 | 1·254 |
| ∞ | 3·841 | 2·996 | 2·605 | 2·372 | 2·214 | 2·099 | 2·010 | 1·938 | 1·831 | 1·752 | 1·517 | 1·000 |

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